A solution in 3 dimensions for current in a semiconductor under high level injection from a point contact

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The standard equations for semiconductor device analysis were solved by specifying the electron and hole current injected at a small contact, assuming high-level injection. Calculated current-voltage characteristics were fit to measurements of a single point breakdown in an ultrathin dielectric. It was found that the minority carrier injection level was about 70%.

Keywords: minority carrier injection, point breakdown, ultrathin oxide

INTRODUCTION

The nature of the contact to a semiconductor at point breakdowns in ultrathin oxides, is not well understood. An important observed feature of these contacts is that they can have a high minority carrier injection ratio. There have been a number of attempts to model point breakdowns in ultrathin oxides on silicon [1]. These models concentrate on the physics at the contact interface and attempt to derive the current vs voltage (*IV*) characteristic. However, in many physical devices the main contribution to the *IV* characteristic is due to the conduction in the bulk of the semiconductor. Therefore, one should include a model of the bulk conductivity in an explanation of point contact behavior. In this paper, such a model is developed using standard semiconductor equations solved for point contacts with minority carrier injection.

POINT CONTACT PHYSICS

This problem has the same geometry as the spreading resistance problem. The difference is that the spreading resistance problem assumes an ohmic contact and only majority carrier conduction. In the point contact problem arbitrary levels of minority carrier injection will be included. This calculation will assume high-level injection, which implies that there is charge neutrality in the bulk. This means that the number of excess holes (beyond the equilibrium concentration) is equal to the number of excess electrons. Since the bulk is neutral, the Laplace equation can be used to find the electric field,

$$\nabla \cdot \mathbf{E} = 0. \qquad (1)$$

The remaining equations needed to describe conduction in a semiconductor are the continuity equations for holes and electrons along with expressions for the hole and electron current in terms of drift and diffusion [2, 3]. For high-level injection, as in the case of high forward bias of a pn junction [2], these equations reduce to

$$\nabla^2 p_t - \frac{p}{L_p^2} = 0, \quad \nabla^2 n_t - \frac{n}{L_n^2} = 0, \qquad (2)$$

for holes and electrons. p_t (n_t) is the total hole (electron) concentration, p (n) is the excess hole (electron) concentration, and p_0 (n_0) is the equilibrium hole (electron) concentration, so that $p_t = p + p_0$ ($n_t = n + n_0$). L_p is the high-level diffusion length, $L_p^2 = 2\mu_n D_p \tau_p / (\mu_n + \mu_p)$, where μ_p and μ_n are hole and electron mobilities, D_p is the hole diffusion constant, and τ_p is the hole lifetime. A similar equation can be written for L_n. Since $\mu_p D_n = \mu_n D_p$ and $\tau_p = \tau_n$, then $L_p = L_n = L$.

These equations must be solved for the boundary conditions of the contact. A semi-infinite structure will be used, where the surface of the semiconductor is on the *x-y* plane and the bulk of the semiconductor extends in the positive *z* direction to infinity. The potential at infinity will be taken as 0. There are mixed boundary conditions on the *x-y* plane. Similar mixed boundary conditions, for the Laplace equation, are used in the solution of the usual spreading resistance problem [4, 5]. Outside the contact radius *a*, the current through the surface and the perpendicular component of the electric field are 0. The boundary conditions on the contact are given by specifying the hole and electron currents through the contact. These conditions are set on the semiconductor side of the contact in order to avoid the physics of the contact interface.

3-D SOLUTION

The problem is cylindrically symmetric about the *z*-axis which is perpendicular to the center of the contact. Equation (2) can be written (ignoring the θ dependence) in cylindrical coordinates,

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} - \frac{p}{L^2} = 0. \qquad (3)$$

This is the Helmholtz equation. It can be solved by separating variables. Using the boundary condition that the potential is zero for *r*, *z* at infinity, the solution for excess hole concentration is

$$p(r, z) = \int_0^\infty A(\lambda) J_0(\lambda r) \exp\left(-[\lambda^2 + 1/L^2]^{\frac{1}{2}} z\right) \mathrm{d}\lambda. \qquad (4)$$

J_0 is a Bessel function of order 0. The function $A(\lambda)$ must be found using the remaining boundary conditions. A similar solution for the electron density can be written.
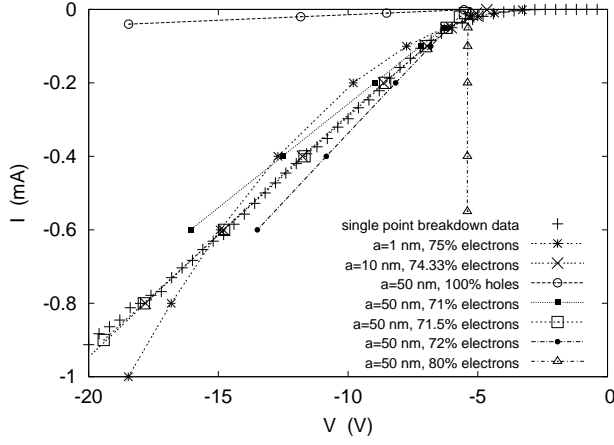
FIG. 1: *IV* data of a single point breakdown. Theory curves have voltage offsets of -2.8 V, -4.5 V, and -5.2 V for $a = 1$ nm, $a = 10$ nm, and $a = 50$ nm, respectively.

An analytic solution can be found if $A(\lambda)$ is chosen to be

$$A(\lambda) = B \frac{J_1(a\sqrt{\lambda^2 + c^2})}{(\lambda^2 + c^2)}\lambda, \qquad (5)$$

where J_1 is a Bessel function of order 1, B is a constant, and $c^2 = 1/L^2$. With this choice, the current density over the area of the contact is approximately constant for $a \ll L$. The electric field at the contact must be consistent with the current, which implies that the on the area of the contact the z component of the field is

$$E_z = E J_0(c\sqrt{a^2 + r^2}), \quad \text{for } r < a \qquad (6)$$

where E is a constant. Since, in the high-level injection case, $p = n$, the solutions can be equated and values of B and E determined for any given values of total hole current, I_h, and electron current, I_n. The potential at the center of the contact can be found from the solution of the Laplace equation and thus the I vs V curve can be calculated. The numerical results below use typical values of parameters for silicon.

Injection of electrons

An interesting case is that of injecting a current of pure electrons into a p-type silicon. In the case of large injected current, the solution for E and the resistance can be approximated as

$$E = \frac{-D_n}{2aG\mu_n}, \qquad R = \frac{0.85D_n}{2G\mu_n I_n}. \qquad (7)$$

G is somewhat dependent on a, but is approximately equal to $4/(3\pi)$. It is seen that as the electron current gets large, the electric field coefficient tends to a constant. This is opposed to the normal Ohm's law behavior where the electric field is proportional to current. The resistance of the contact actually decreases as $1/I_n$ as opposed to the ohmic behavior of a constant resistance.

Single point breakdown

A single point breakdown device was fabricated by opening a 100 nm square window in a thick (20 nm) SiO$_2$ insulating layer on a 6 Ω·cm p-type silicon wafer. A thin SiO$_2$ layer (2 nm) was grown in this opening and an indium tin oxide top electrode was deposited on top of this [6]. The device was voltage stressed to create a hard breakdown. IV data of this breakdown site is presented in Figure 1. Theoretical *IV* curves were calculated assuming a constant electron injection ratio. A constant voltage was added to model the voltage drop across the contact interface for large current. The experimental current is more than 10 times larger than the calculated current for only hole injection. Considering the curvature at low currents, the best fit is for $a = 10$ nm and an electron injection of 74%. With smaller values of a the theoretical curve is not straight and cannot fit the measured data. A reasonable fit can be obtained for a range of contact size, however it is clear that the electron injection ratio is between 70% and 75%. This demonstrates that a large minority injection ratio exists for these devices.

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